GAG—2009. Gymn.—1st year

1. Pole ***(10 points)*** The energy of the puck conserves, so its velocity is constant and the overall time is found as $t = L/v$, where L is the length of the trajectory. The trajectory is made up of 90° circular segments of decreasing radius (the first segment is 135°), n-th radius is $R_n = l - np/4$. So, the trajectory length is $L = \frac{\pi}{2}(R_0 + R_1 + \ldots + p/4 + R_0/2) = \frac{\pi}{4}(4R_0^2/p + R_0) = \pi(R_0^2/p + R_0/4)$. Finally, $t = \pi(R_0^2/p + R_0/4)/v = 10.5\pi$ s≈ 33 s.

2. Glass ***(10 points)*** The water level difference inside and outside the glass is $h = 22$ cm (bearing in mind the scale factor 10). So, $\Delta p = \rho g h = 2.2$ kPa, and $p = p_0 + \Delta p = 102.2$ kPa.

As for the mass, we need to calculate the volume V of the system "glass+air inside the glass" under the exterior water level. $V = \pi[R^2H - r^2(H-h)] = \pi[H(R^2-r^2)+hr^2]$, where $R = 22$ cm and $r = 16$ cm are the external and internal radii of the glass and $H = 33$ cm — its depth under the water level. So, $V \approx 41$ l, and hence, $m \approx 41$ kg.

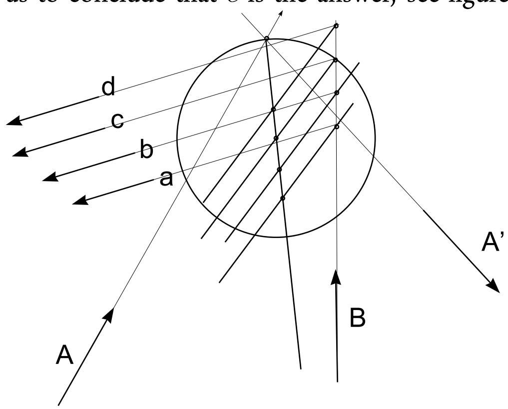
Note that the competition papers had a slightly enlarged figure (as compared to the web version); so, all the linear dimensions of the examination papers should have been ca 1.3 times larger than here (and the mass 1.3^3 times).

3. Ball ***(10 points)*** In the system of reference of the centre of mass, the approaching speed and departing speed are equal (for both bodies). So, we use the system which moves with velocity $w = \frac{1}{2}(u_0 + v_0) = 50$ km/h. In that system, the ball approaches and departs with the velocity $v = v_0 + w$, and, hence, in the laboratory frame, departs with the velocity $v_1 = v + w = v_0 + 2w = 190$ km/h.

Here we assumed that the velocities u_0 and u_1 are unidirectional; taking u_1 antiparallel to u_0 would result in $v_1 = 110$ km/h. However, anyone who has seen how tennis is played would probably exclude the possibility that the racket bounces back with almost the same speed as the hitting speed.

4. Tetrahedron ***(10 points)*** Neglecting the internal resistance of the ammeter, the circuit has two parallel connections, which connected in sequence. The reading of the ammeter is the difference of the currents I_1 and I_2 in the lower (or upper) branches of the parallel connections. Due to symmetry, both parallel connections have applied voltage $U = \mathcal{E}/2$; hence the two currents are $I_1 = \mathcal{E}/2R_1$ and $I_1 = \mathcal{E}/2R_2$. So, the reading of the ammeter is $I = \frac{1}{2}\mathcal{E}(R_1^{-1} - R_2^{-1}) = 0.5$ A.

5. Concave mirror ***(10 points)*** The centre of the mirror lies on the cross-section point of the bisectors of the angles formed by the beams A and A' on the one hand, and B and B' on the other hand. The reflection points (i.e. the intersection points of the incident and reflected beams) must be on the equal distance from the centre. This condition allows us to conclude that c is the answer, see figure.



GAG—2009. Gymn.—2nd year

1. Pole ***(10 points)*** The trajectory is made up of 90° circular segments of decreasing radius (the first segment is 135°), n-th radius is $R_n = l - np/4$. So, the trajectory length is $L = \frac{\pi}{2}(R_0 + R_1 + \ldots + p/4 + R_0/2) = \frac{\pi}{4}(4R_0^2/p + R_0) = \pi(R_0^2/p + R_0/4) = 10.5\pi$ m≈ 33 m. The kinetic energy of the puck dissipates as the work of the friction forces $v_0^2 = 2\mu g L$. So, $v_0 = \sqrt{2\mu g L} \approx 8$ m/s.

A pole of square cross-section is fixed vertically to a flat horizontal surface. The perimeter of the pole is $p = 40$ cm. A rope of length $l = 2$ m is fixed to the corner of the pole, at the ice surface. A small puck is fixed to the other end of the rope. Initially, the rope is horizontal and stretched so that the line of the rope passes through the centre of the pole; the puck rests on the ice. Which horizontal initial velocity (perpendicular to the rope) has to be given to the puck, such that the rope would get completely winded around the pole? The rope is much lighter than the puck; the coefficient of friction between the puck and the surface is $\mu = 0.1$.

2. Glass ***(10 points)*** The water level difference inside and outside the glass is $h = 22$ cm (bearing in mind the scale factor 10). So, $\Delta p = \rho g h = 2.2$ kPa, and $p = p_0 + \Delta p = 102.2$ kPa.

As for the mass, we need to calculate the volume V of the system "glass+air inside the glass" under the exterior water level. $V = \pi[R^2H - r^2(H-h)] = \pi[H(R^2-r^2)+hr^2]$, where $R = 22$ cm and $r = 16$ cm are the external and internal radii of the glass and $H = 33$ cm — its depth under the water level. So, $V \approx 41$ l, and hence, $m \approx 41$ kg.

Note that the competition papers had a slightly enlarged figure (as compared to the web version); so, all the linear dimensions of the examination papers should have been ca 1.3 times larger than here (and the mass 1.3^3 times).

3. Ball ***(10 points)*** In the system of reference of the centre of mass, the approaching speed and departing speed are equal (for both bodies). So, we use the system which moves with velocity $w = \frac{1}{2}(u_0 + u_1) = 50$ km/h. In that system, the ball approaches and departs with the velocity $v = v_0 + w$, and, hence, in the laboratory frame, departs with the velocity $v_1 = v + w = v_0 + 2w = 190$ km/h. The ratio of masses is given by the ratio of velocities in the center of mass system; so, $M/m = v/(w - v) = 14$.

Note that real tennis rackets have typically mass between 250 g and 400 g; the mass of the tennis ball is 57 g. So, in this problem, either the ball was not a real tennis ball, or the racket was very heavy.

Here we assumed that the velocities u_0 and u_1 are unidirectional; taking u_1 antiparallel to u_0 would result in $v_1 = 110$ km/h and $M/m = 2$. However, anyone who has seen how tennis is played would probably exclude the possibility that the racket bounces back with almost the same speed as the hitting speed.

4. Tetrahedron ***(10 points)*** Neglecting the internal resistance of the ammeter, the circuit has two parallel connections, which connected in sequence. The reading of the ammeter is the difference of the currents I_1 and I_2 in the lower (or upper) branches of the parallel connections. Due to symmetry, both parallel connections have applied voltage $U = \mathcal{E}/2$; hence the two currents are $I_1 = \mathcal{E}/2R_1$ and $I_1 = \mathcal{E}/2R_2$. So, the reading of the ammeter is $I = \frac{1}{2}\mathcal{E}(R_1^{-1} - R_2^{-1}) = 0.5$ A.

5. Concave mirror ***(10 points)*** The centre of the mirror lies on the cross-section point of the bisectors of the angles formed by the beams A and A' on the one hand, and B and B' on the other hand. The reflection points (i.e. the intersection points of the incident and reflected beams) must be on the equal distance from the centre. This condition allows us to conclude that c is the answer, see figure (in the second column).

GAG—2009. Gymn.—3rd year

1. Magnetic field ***(10 points)*** The electron moves along a circle in the magnetic field (curvature direction is defined by the screw law), and along ´a straight line outside. The resulting sketch is given below. As can be seen from the figure, the particle draws a $\frac{1}{2} - \alpha/\pi$ fraction of the circle (for $\frac{\pi}{2} > \alpha > -\frac{\pi}{2}$); so, the time spent inside the magnetic field is $T = (\pi - 2\alpha)\frac{m}{eB}$.

2. Glass ***(10 points)*** The water level difference inside and outside the glass is $h = 26.5\,\mathrm{cm}$ (bearing in mind the scale factor 10). So, $\Delta p = \rho g h = 2.65\,\mathrm{kPa}$, and $p = p_0 + \Delta p = 102.7\,\mathrm{kPa}$.

As for the sinking of the glass, the air volume inside the glass needs to be decreased by the amount, equal to the volume of the glass, protruding above the water level, $V_a = \pi R^2 a \approx 3.01$, where $a = 2$ cm is the height of the glass above the water level, and $R = 22$ cm — its external radius. The air volume inside the glass is $V_i = \pi r^2 H \approx 36\,1$, where $r = 20.5$ cm is the internal radius of the glass and $H = 27$ cm — the air column height inside the glass. Since the water columns contribute just a tiny fraction to the air pressure, we can neglect the change of the air pressure due to sinking. Therefore, we can use the isobaric law: $\Delta T \approx TV_a/V_i \approx 26\,\mathrm{K}$. So, the glass sinks at $T = t - \Delta T \approx 4°\mathrm{C}$.

Note that the competition papers had a slightly enlarged figure (as compared to the web version); so, all the linear dimensions of the examination papers should have been ca 1.3 times larger than here.

3. Brick ***(10 points)*** Since $mg \ll T$, we can neglect the gravity. So, $T = m\omega^2 R$, from where $\omega = \sqrt{T/mR} \approx 100\,\mathrm{rad/s}$. The trajectory is a straight line, tangent to the shaft (at 50 cm, the curvature due to the gravity is negligible).

4. Tetrahedron ***(10 points)*** Since the charges on the capacitors are equal (due to the conservation of the net charge: two plates are connected and have zero net charge), the ratio of voltages is equal to the inverse ratio of the capacitances. So, the capacitances C_1 have voltages $U_1 = \mathcal{E}C_2/(C_1 + C_2)$, and the capacitances C_1 have voltages $U_2 = \mathcal{E}C_1/(C_1 + C_2)$. The reading of the voltmeter is the difference of these two; so $U = \mathcal{E}(C_2 - C_1)/(C_1 + C_2) \approx 0.67\,\mathrm{V}$.

5. Convex mirror ***(10 points)*** The centre of the mirror lies on the cross-section point of the bisectors of the angles formed by the beams A and A' on the one hand, and B and B' on the other hand. The reflection points (i.e. the intersection points of the incident and reflected beams) must be on the equal distance from the centre. This condition allows us to conclude that b is the answer, see figure.