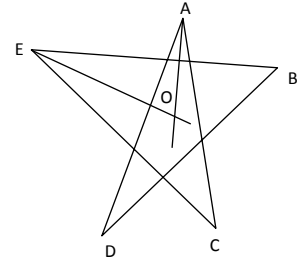


Form 11

1. Let ABCDE be five-sided star (the length of the sides are not necessarily equal). It is given that all the angles „at the vertices” are the same (for example $\angle ADB = \angle EBD$). Prove that the bisectors of any two consecutive vertex angles intersect in 72° angle. (In example angle $\angle EOA$ have to be 72°)



2. Find the polynomial P with degree 3, such that $P(1) = 3$, $P(2) = 0$, $P(3) = 1$.
3. Function $g(x)$ is defined for real x as $g(x) = |x|$, if x is a whole number and as $g(x) = -\frac{1}{x}$, if x is not a whole number. Solve the inequality $g(x - 1) < ax - a$, where a is given real positive number.
4. Given that $a = (\sqrt{2})^x$. For which rational values of a, rational solution x exists?
5. Consider n houses built in vertices of n-gone, where each house has a number from 1 to n. For Christmas each of the n houses was given to one of the n dwarfs. To make it more fun, if the dwarf was assigned to the house with number k, it went to live in the house with number k^2 (counting from the first house and continuing to count for several circles if necessary). Prove that: 1) if there is an odd number of dwarfs and there is exactly one lonely dwarf, it lives in the last house. 2) If there are exactly two lonely dwarfs, then they live as far from each other as possible.

Notes: After settling in each house can be inhabited by as many dwarfs as necessary. Lonely dwarfs are dwarfs that are living in the house alone.