



Mathematics Form 12



English

1. A coin is tossed a number of times, where the two possibilities “head” and “tail” both have the probability $1/2$ of occurring. A and B play the following game. A wins as soon as the combination head, head, tail directly after each other occurs, while B wins as soon as the combination tail, head, tail directly after each other occurs. What is the probability that A wins?
2. Assume that a and b are positive whole numbers such that the equation $ax + by = c$ has whole number solutions x, y for all whole numbers c . Show that if $c > ab - a - b$, then the equation $ax + by = c$ has solutions where x, y are non-negative whole numbers.
3. Two lines intersect each other at an angle of 7° forming a funnel. A beam of light entering this funnel strikes one side of the funnel at an angle of 5° and is reflected at an angle of 5° (according to the law of reflectance). The beam then strikes the other side of the funnel and is reflected in the same manner back towards the first side. How many times in total is the beam of light reflected from the two sides before it leaves the funnel.
4. Show that there are whole numbers x, y such that $x^2 - y^2 = a$ if and only if a is an uneven whole number or a is a whole number which is divisible by 4.
5. a) A series of whole numbers $a_1, a_2, a_3 \dots$ is such that $a_1 = 1, a_2 = 3$ and a_n for $n > 2$ is equal to the sum of the two preceding members of the series, e.g. $a_n = a_{n-1} + a_{n-2}$. Show that no member of the series is divisible by 10.
b) A series of whole numbers (known as the Fibonacci series) $b_0, b_1, b_2 \dots$ is defined by $b_0 = 0, b_1 = 1$, and $b_n = b_{n-1} + b_{n-2}$ for $n > 1$. Show that there is a value of n such that b_n is divisible by 1000.