Physics Solutions

Form 10

Task 1
1a

Force equilibrium upper ball, case 1 $$mg = k\frac{6q^2}{0.05^2}$$

Force equilibrium upper ball, case 2 $$Mg = k\frac{6q^2}{0.03^2}$$

This gives $$\frac{M}{m} = \left(\frac{0.05}{0.03}\right)^2 = \frac{25}{9}$$

Correct equilibrium equations: 1 point

Correct solution of the system: 2 points

If the dependence 1/r is used instead of 1/r^2 in Coulomb's law and the remainder is correct so that the answer is given as M/m = 5/3: a total of 1 point

1b
The charges are equalised to Q on each ball $$Q = \frac{6q+q}{2} = 3.5q$$

Equilibrium upper ball, case 3 $$Mg = k\frac{3.5^2q^2}{x^2}$$

Comparison, cases 2 and case 3 $$Mg = k\frac{3.5^2q^2}{x^2} = k\frac{5q^2}{0.03^2}$$

This gives $$x = 0.03\cdot 3.5\sqrt{\frac{1}{5}} = 4.7\text{ cm}$$

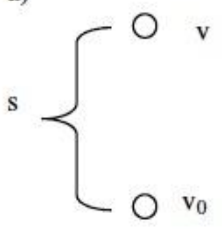
The charge Q and equilibrium in case 3 correctly expressed: 1 point

Correct equation for x: 1 point

Correct solution of the equation: 1 point

Form 10 –Task 2

a)



$$v^2 - v_0^2 = -2gs \quad \Rightarrow \quad v_0 = \sqrt{2gs} = \sqrt{2 \cdot 9.82 \cdot 1.8} = 5.95 \text{ m/s}$$

Correct expression for v_o: 1 point
Correct solution for v_o: 1 point

b)

$$v = v_0 - gt \quad \Rightarrow \quad t = \frac{v_0}{g} = \frac{5.95}{9.82} = 0.606 \text{ s}$$

Correct expression for t: 1 point
Correct solution for t: 1 point

c)
Total time for one lap: $2 \cdot 0.605 + 3 = 1.51$ s

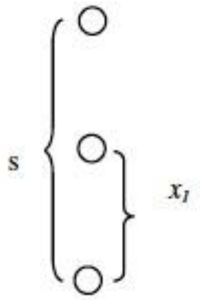
Distance in time between balls= 1.51/3 = 0.503 s

The other two oranges are at the same distances in time from the orange at the highest point. One orange is on its way down and is then at a distance $y = 1.8 – (9.82 \cdot 0.503^2)/2 = 0.55$ m above the hand. The third orange is on its way up and is then also at a distance $y = 1.8 - (9.82 \cdot 0.503^2)/2 = 0.55$ m above the hand (the same distance in time from the top position as the second orange).

(for the third orange, y = 1.8 – distance from the orange at the highest point
$= 1.8 – (vt – gt^2/2)) = 1.8 - (gt^2 - gt^2/2) = 1.8 – gt^2/2$
[since v = gt is the magnitude of the orange’s upward speed]

Correct distance for one orange: 1 point
Correct distance for the other orange: 1 point

Alternative solution:

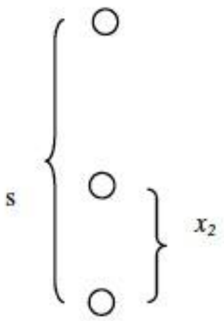


Orange No 2 on its way down. Free fall gives:

$$s - x_1 = \frac{gt^2}{2}$$

$$x_1 = s - \frac{gt^2}{2} = 1.8 - \frac{9.82 \cdot 0.503^2}{2} = 0.55 \text{ m}$$

Time for orange 3 after it left the hand: 0.503 – 0.3 – (0.605 – 0.503) = 0.101 s



$$x_2 = v_0\, t - \frac{gt^2}{2} = 5.95 \cdot 0.101 - \frac{9.82 \cdot 0.101^2}{2} = 0.55 \text{ m}$$

Correct distance for one orange: 1 point
Correct distance for the other orange: 1 point

Form 10 –Task 3

a) The energy principle gives

$$\tfrac{1}{2}mv_A^2 + mgh = \tfrac{1}{2}mv_B^2 + E_v \,.$$

The heat of friction is: $E_v = mgh$ = 15.7 J (constant speed down the ramp!)

Correct formulation of energy principle: 1 point
Correct calculation of E_v: 1 point

b) Calculation of (magnitude of) the frictional work W_f

$$E_v = |W_f| = \sqrt{3}/2\, F\ \overline{AB} \Rightarrow 15.7 = \sqrt{3}/2\, F\ 1.6 \Rightarrow$$

$$F = 11.3 \text{ N}$$

Work = force times distance: 1 point
Correct calculation of F: 1 point

c) The energy principle gives

$$\tfrac{1}{2}mv_B^2 = \tfrac{1}{2}mv_C^2 + Fx$$

The box stops at a distance x = 1.42 m from B

Correct formulation of energy principle: 1 point
Correct calculation of x: 1 point

Form 10 –Task 4

a) The area of the base: $A = (1.5R)^2\ \pi = 2.25\pi R^2$

Volume of water: $V_v = h \cdot A$

Volume of water displaced, half the volume of the ball: $V = 2\pi R^2/3$

Total volume under the surface in the right-hand figure can be written in two ways since the water volumes are equal:

$$V_{total} = V_v + V = (h + x)A$$

$$x = \frac{V}{A} = \frac{2\pi R^3}{3} \cdot \frac{1}{2.25\pi R^2} = \frac{8}{27}R = 0.296R$$

Correct value of x: 1 point

b)

Increase in pressure on the base $\Delta p = \rho_v g(h + x) - \rho_v gh = \frac{8}{27}\rho_v gR = 2910R$

Alternative solution:

Δp = Increase in weight/ area of base = $\frac{\rho_{boll} \cdot V_{sfär} \cdot g}{2.25\pi R^2} = \frac{500 \cdot 4\pi R^3}{3 \cdot 2.25\pi R^2} g = 2910R$

Correct method but incorrect value of x: 1 point
Correct answer: 2 points

c)

The water rises a distance x for all $h \geq h_{min}$

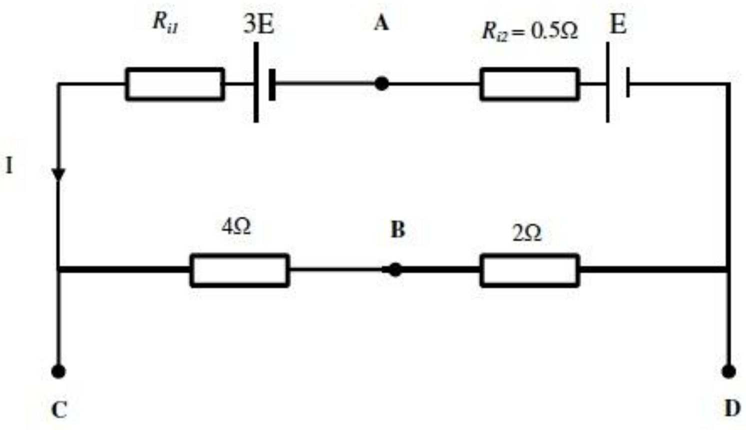
$$h_{\min} = R - x = \frac{19}{27}R = 0.704R$$

Correct answer: 2 points

Form 11

Form 11 –Task 1

a) Introduce the current I



$$I = \frac{3E + E}{R_{i1} + 4 + 2 + 0.5} = \frac{4E}{R_{i1} + 6.5}$$

(1 point)

If a short-circuiting between A and B does not affect the potential between C and D, then U_{AB} = 0 and the current *I* is unchanged, i.e.:

$$U_{AB} = 0$$

$$3E - I(R_{i1} + 4) = 0 \;\Rightarrow\; I = \frac{3E}{R_{i1} + 4}$$

$$I = I \quad \Rightarrow \quad \frac{4E}{R_{i1} + 6.5} = \frac{3E}{R_{i1} + 4}$$

$$\frac{4}{R_{i1} + 6.5} = \frac{3}{R_{i1} + 4}$$

(1 point)

$$4(R_{i1} + 4) = 3(R_{i1} + 6.5)$$
$$4R_{i1} + 4 \cdot 4 = 3R_{i1} + 3 \cdot 6.5$$
$$R_{i1} = 3 \cdot 6.5 - 4 \cdot 4 = 3.5\ \Omega$$

(1 point)

Answer: R_{i1} = 3.5 *Ω*

b)

The potential difference between A and D is given by

$$0 + E - R_{i2}I = V_A$$

(1 point)

$$I = \frac{3E}{R_{i1} + 4} = \frac{3E}{3.5 + 4} = \frac{3E}{7.5} = 0.4E$$

(1 point)

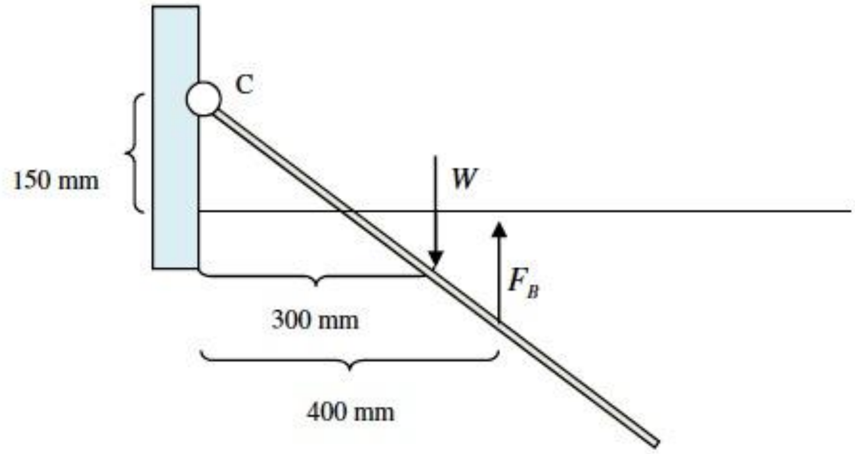
$$V_A = E - 0.5 \cdot 0.4E = 0.8E$$

(1 point)

Answer: The electrical potential at A is 0.8 *E*

Form 11 –Task 2

a)



According to Pythagoras, the length of the rod is: $L = 0.75$ m

Cross-section area of the rod: A

Moment balance about C: $0.4 \cdot F_B = 0.3W$, so that

Archimedes buoyancy (m = 1 kg) : $F_B = 0.75g$ (1)

Volume of water displaced: $V_{under} = 0.5A$

Archimedes buoyancy: $F_B = \rho_v V_{under} g = 500gA$ (2)

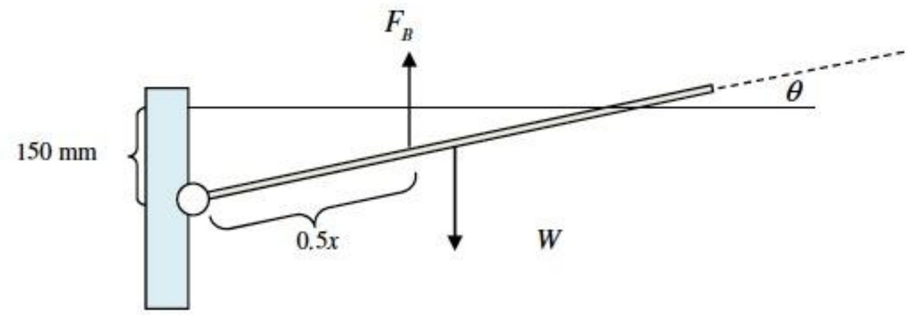
The same force in (1) and (2) gives the area: $A = 0.75/500 = 0.0015$ m^2

Density of the rod: ρ_{wood} = mass/volume = $\frac{1}{LA} = \frac{500}{0.75^2} = \frac{8}{9} \cdot 1000$ kg/m^3

Correct answer by some other correct method: 3 points

Using his method,
Correct forces at the correct positions: 1 point
Correct balance of moments about C: 1 point
Correct density: 1 point

b)



Balance of moments about *C*:

$$F_B \cdot 0.5x\cos\theta = W \cdot 0.5L\cos\theta$$
$$\rho_v Axg \cdot 0.5x\cos\theta = \rho_{wood} ALg \cdot 0.5L\cos\theta$$
$$\rho_v \cdot x^2 = \rho_{wood} \cdot L^2$$
$$x = \sqrt{\frac{\rho_{wood}}{\rho_v}} \cdot L = \sqrt{\frac{8}{9}} \cdot 0.75 = 0.707 \text{ m}$$

Correct answer by some other correct method: 3 points

Using his method,
Correct forces at the correct positions: 1 point
Correct balance of moments about C: 1 point
Correct length x : 1 point

Form 11 –Task 3

a) The energy E released when two stones each of mass m and heat capacity c_s heat up the water with mass m_{H20} and heat capacity c_{H20} so that the temperature increases from 20°C to 40° C (assuming no losses) is:

$$E = 2m_s c_s (200-40) = m_{H_2O} \cdot c_{H_2O}(40-20) \qquad (1)$$

In the same way, we obtain that the three stones heat the water to T°C

$$3m_s c_s (200-T) = m_{H_2O} \cdot c_{H_2O}(T-20) \qquad (2)$$

Equation (1) gives: $$\frac{m_s c_s}{m_{H_2O} \cdot c_{H_2O}} = \frac{1}{16} \qquad (3)$$

Equation (2) leads to: $$3 \cdot \frac{1}{16}(200-T) = T-20$$

The answer is $$T = \frac{920}{19} = 48.4^0\text{C}$$

Correct equation (1): 1 point
Correct equation (2); 1 point
Correct answer: 1 point

b)

Assume that x stones give exactly 90 °C, but that x need not be a whole number

$$x \cdot m_s c_s (200-90) = m_{H_2O} \cdot c_{H_2O}(90-20)$$

Equation (3) then gives:

$$x = 16 \cdot \frac{70}{110} = 10.2$$

The answer is therefore that, for the water temperature to reach 90°C, 11 whole stones are required

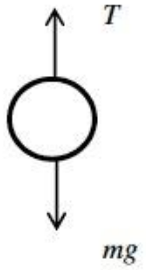
Correct equation: 1 point

Correct answer: 2 points

Form 11 –Task 4

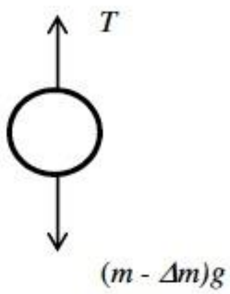
Isolate the balloon and consider Newton's second law.

The acceleration *a* downwards



$$mg - T = ma \qquad 1)$$

The acceleration *a* upwards



$$T - (m - \Delta m)g = (m - \Delta m)a \qquad 2)$$

Understood that Newton's second law in the vertical direction must be used twice: 1 point
Equations (1) and (2) but without the lifting force: 1 point
Equations (1) and (2) but with incorrect sign: 1 point
Correct formulation of equations (1) and (2): 3 points
Total points for this part: maximum 3 points

Take e.g. equations (1) and (2) in order to eliminate the lifting force *T*, which gives a relationship which can be solved for Δm

$$1) \; +2) \quad \Rightarrow \quad mg - mg + \Delta mg = 2ma - \Delta ma \qquad 3)$$

$$\Rightarrow \qquad \Delta m = 2m\frac{a}{g+a}$$

The numerical solution is:

$$\Delta m = 800\frac{0.01\,g}{g(1+0.01)} = 7.92\text{ kg} \approx 8\text{ kg}$$

Correct solution of the equation system: 2 points
Correct numerical solution: 1 point

Form 12

Form 12 –Task 1

Equivalent resistance:

$$R_e = \frac{20R}{20+R}$$

The current according to Ohm's law:

$$I = \frac{12}{2+R_e} = \frac{12}{2+\dfrac{20R}{20+R}} = \frac{12(20+R)}{2(20+R)+20R} = \frac{240+12R}{40+22R}$$

The potential across R_e :

$$V_{AB} = 12 - 2I = 12 - 2\cdot\frac{240+12R}{40+22R} = \frac{12(40+22R) - 480 - 24R}{40+22R} = \frac{240R}{40+22R}$$

The power developed in R is given by:

$$P(R) = \frac{V_{AB}^2}{R} = \frac{240^2 R}{(40+22R)^2}$$

Correct answer but not simplified: 3 points
Correct expression for current simplified as far as possible: 1 point
Correct potential across R simplified as far as possible: 1 point
Correct expression for the power simplified as far as possible: 2 points
Total maximum: 4 points

b)

The power is zero for R = 0, and R⇨ ∞ therefore has a maximum at $dP/dR = 0$

$$0 = \frac{1}{(40+22R)^2} - \frac{2\cdot 22R}{(40+22R)^3}$$

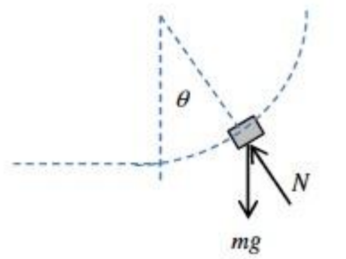
This leads to: $40 + 22R - 44R = 0$

$$R = \frac{40}{22} = 1.82\,\Omega$$

Shown that an extreme value is obtained and that this is a maximum: 1 int
Correct answer: 3 points
Total maximum: 4 points

Form 12 –Task 2

a) Consider the puck in an arbitrary position in the circular track and use Newton's second law.



$$\sum F = ma_c \quad \Rightarrow \quad N - mgcos\theta = m\frac{v^2}{r} \quad \Rightarrow \quad N = mgcos\theta + m\frac{v^2}{r} \qquad 1)$$

It is evident that the normal force *N* is greatest when θ □ □□ □□□□□□□□□□□θ □ □□, which means that the whole of *mg* contributes to the normal force *N*. This occurs when θ □ □, □□□□□□immediately after the puck passes position *B*.

Only correct reasoning (even without equations showing how gravitational force affects the normal force: 1 point
Only equation (1), but correct expression for centripetal acceleration: 1 point
Equation (1) and the correct conclusion that N is greatest when $\theta = 0$: 2 points
Total maximum: 2 points

b) Equation (1) with $N = 10mg$ and θ □ □□ gives

$$10mg = mg + m\frac{v_B^2}{r} \quad \Rightarrow \quad v_B = 3\sqrt{rg} = 6.65 \text{ m/s}$$

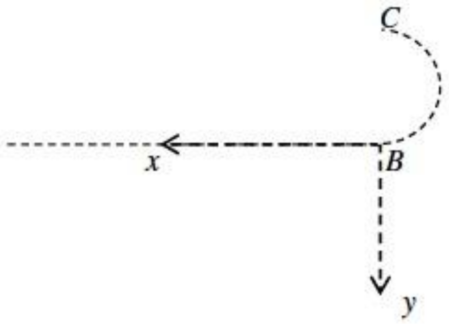
Correct equation: 1 point
Correct calculation of v_B: 1 point
Total maximum: 2 points

c) The energy principle gives.

$$2mgr + \frac{1}{2}mv_C^2 = \frac{1}{2}mv_B^2 \quad \Rightarrow \quad v_C = \sqrt{5rg} = 4.95 \text{ m/s}$$

Correct formulation of the energy principle: 1 point
Correct calculation of v_C: 1 point
Total maximum: 2 points

d) Consider an x-y coordinate system with its origin at *B* as in the diagram below:



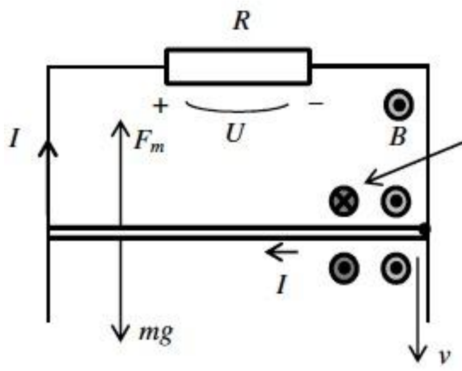
The trajectory is then described by the following equations:

$$v_x = v_c \qquad 1)$$
$$x = v_c t \qquad 2)$$
$$v_y = gt \qquad 3)$$
$$y = -2r + \frac{1}{2}gt^2 \qquad 4)$$

$$4) \Rightarrow 0 = -2r + \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{4r}{g}}$$
$$2) \Rightarrow x = v_c\sqrt{\frac{4r}{g}} = \sqrt{5rg}\sqrt{\frac{4r}{g}} = r\sqrt{20} \; = 2.24 \text{ m}$$

Correct formulation of the trajectory: 1 point
Correct solution of the equations : 1 point
Total maximum: 2 points

Form 12 –Task 3



A suitable explanation of why the field is in this direction is required: 2 points

a)

An explanation related to the current direction is required, see above: 2 points

$$U = Blv$$
$$I = \frac{U}{R} = \frac{Blv}{R}$$

2 points
Can be used without sign discussion

b)

$$F_m = BIl$$
$$F_g = mg$$
$$F_m = F_g$$

1 point – Justification for this is essential

$$mg = BIl$$

1 point

$$mg = B\frac{Blv_{slut}}{R}l$$

$$mg = \frac{B^2l^2\,v_{slut}}{R}$$

$$v_{slut} = \frac{mgR}{B^2l^2}$$

2 points

Answer: The constant speed is

$$v = \frac{mgR}{B^2l^2}$$

Form 12 –Task 4

$$mg = (1.0 + 0.2)g = 11.78 \text{ N}$$

Both springs are elongated to the same extent (coupled in parallel).

a) Figure 2 gives the spring constant:

$$k = 6/0.1 = 60 \text{ N/m.}$$ *1 point*

The elongation in the equilibrium position is

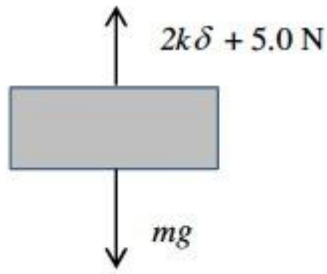
$$\delta = \frac{mg}{2k} = \frac{11.78}{120} = 0.0982 \text{ m}$$ *1 point*

b) When the weight is subjected to the force F = 5.0 N, the springs are elongated further

$$s = \frac{F}{2k} = \frac{5}{120} = 0.0417 \text{ m.}$$ *1 point*

This is the amplitude of the oscillation, which is thus 0.0417 m *1 point*

c) If F = 5.0 N, the weight is subjected to the following forces immediately after the string is cut:



The spring force directed upwards which gives the maximum acceleration in the upward direction is then 5.0 N. *1 point*

and $F = ma$ gives a = 5.0 N/1.2 kg = 4.17 m/s^2 *1 point*

d) B loses contact with A in the uppermost position if the springs are relaxed in that position so that the weight is influenced only by the gravitational force. *1 point*

Alternatively, A's acceleration in the uppermost position is equal to g and is directed downwards. *1 point*

In that case, F shall be such that the weights are drawn down a further distance δ (see part a)) below the equilibrium position. This occurs if

$$F = mg = 11.78 \text{ N}$$ *1 point*

Total maximum: 2 points for part d)