The 28th International Science Olympiad for Young Mathematicians, Physicists and Chemists
November 3, 2015
Physics solutions - Form 11

1. a) Let the volume of the box be V. Immediately after it's release the box is acted upon by gravity and buoyancy (*1 point*). If we choose up to be the positive direction, then gravity can be written as $F_g = -V\rho_b g$ and buoyancy as $F_b = V\rho_w g$. Newton's second law takes the form $V\rho_b a = V\rho_w g - V\rho_b g$ (*1 point*) or $a = \left(\frac{\rho_w}{\rho_b} - 1\right) g$. From this we can find that $\frac{\rho_w}{\rho_b} = \frac{a}{g} + 1$ and $\rho_b = \frac{g}{g+a}\rho_w = 800\,\mathrm{kg/m^3}$ (*1 point*).

b) The box is floating freely if gravity and buoyancy balance each other. The volume of the submerged part of the box is Vf_1, so $V\rho_b g = Vf_1\rho_w g$ (*1 point*). From this we can see that $f_1 = \frac{\rho_b}{\rho_w} = \frac{g}{g+a} = \frac{4}{5}$ (*1 point*).

c) The volume of the part of the box that is in water is Vf_2 and the volume of the part of the box that is in the other liquid is $V(1-f_2)$. The balance of forces can be written as $V\rho_b g = Vf_2\rho_w g + (1-f_2)V\rho_l g$ (*1 point*). Dividing this by Vg and rearranging gives $\rho_l = \frac{\rho_b - f_2\rho_w}{1-f_2}$ (*1 point*). Substituting ρ_b gives $\rho_l = \left(\frac{g}{g+a} - f_2\right)\frac{\rho_w}{1-f_2} = 750\,\mathrm{kg/m^3}$ (*1 point*).

2. a) Let the more massive box initially move in the positive direction. Then the total momentum of the system before the collision was $p = m_2 v - m_1 v = 2m_1 v$ (*1 point*). After the collision the total momentum was $p = m_1u_1 + m_2u_2$, so from the conservation of momentum we can conclude that $v(m_2 - m_1) = m_1u_1 + m_2u_2$ (*1 point*). Additionally $u_1 - u_2 = v$ and therefore $u_2 = u_1 - v$ (*1 point*). Substituting this to the previous equation we obtain $v(m_2 - m_1) = m_1u_1 + m_2(u_1 - v)$, from which we get $u_1 = \frac{2m_2 - m_1}{m_1 + m_2}v = \frac{5}{4}v = 1.25\,\mathrm{m/s}$ (*1 point*) and $u_2 = \frac{1}{4}v = 0.25\,\mathrm{m/s}$ (*1 point*). We can see that after the collision both boxes move in the direction the more massive box was moving in before the collision.

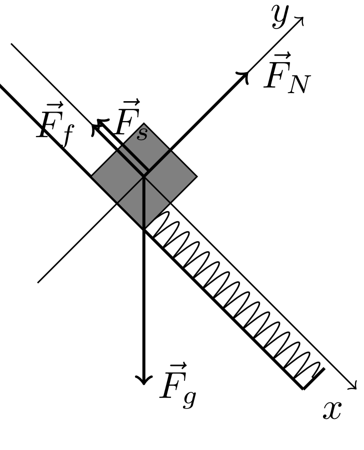
b) The initial kinetic energy of the boxes was $E_0 = \frac{m_1v^2}{2} + \frac{m_2v^2}{2} = 2m_1v^2$ (*1 point*). After the collision it was $E_1 = \frac{m_1u_1^2}{2} + \frac{m_2u_2^2}{2} = \frac{7}{8}m_1v^2$ (*1 point*). The kinetic energy lost in the collision was therefore $E_0 - E_1 = 1\frac{1}{8}m_1v^2 = 1.125\,\mathrm{J}$ (*1 point*).

3. a) Electric current trough the circuit with one resistor was $I_1 = \frac{U}{R+r}$ (*1 point*). The power of the resistor as a heat source was $P_1 = I^2R = \frac{R}{(R+r)^2}U^2$ (*1 point*) and the heat emitted by it was $Q_1 = P_1t_1$. This heat was absorbed by water, so $Q_1 = Mc\Delta T$ (*1 point*). From this we get $\frac{RU^2t_1}{(R+r)^2} = Mc\Delta T$ (*1 point*) and $r = U\sqrt{\frac{Rt_1}{Mc\Delta T}} - R$ (*1 point*). Analyzing the circuit with two resistors would analogously lead us to the equation $r = U\sqrt{\frac{Rt_2}{2Mc\Delta T}} - \frac{R}{2}$. Only one of these equations is needed to answer this question.

b) In order to find the voltage U it is necessary to derive both of the possible answers to the last question (*2 points*). The equality of the right-hand sides $U\sqrt{\frac{Rt_1}{Mc\Delta T}} - R = U\sqrt{\frac{Rt_2}{2Mc\Delta T}} - \frac{R}{2}$ (*1 point*) then follows from the equality of the left-hand sides. We can substitute t_2 (*1 point*) and write $U\sqrt{\frac{Rt_1}{Mc\Delta T}} = \frac{5}{9}U\sqrt{\frac{Rt_1}{Mc\Delta T}} + \frac{R}{2}$ that can be rewritten as $U = \frac{9}{8}\sqrt{\frac{RMc\Delta T}{t_1}} = 315\,\mathrm{V}$ (*1 point*).

c) The numerical value of the internal resistance of the voltage source is $r = 35\,\Omega$ (*2 points*).

4. a) The forces acting on the box are gravity $\vec{F}_g$, normal force $\vec{F}_N$, friction $\vec{F}_f$ and the force applied by the spring $\vec{F}_s$ (*1 point*). Let the x-axis be parallel to the axis of the spring and the y-axis perpendicular to the inclined plane. In that case the y-component of gravity is always balanced by the normal force and we only need to concern ourselves with the forces along the x-axis. In the equilibrium state the x-component of gravity is balanced by friction and the force applied by the spring (*1 point*). Lowering the bottom end of the spring decreased it's compression and also the force it applied to the box. The fact that the box immediately responded to that by starting to move means that the force of friction in the final state was



at it's maximum value (*1 point*). Otherwise increase in friction would have been able to keep the box still, at least initially. Therefore $mg\sin\alpha = \mu mg\cos\alpha + k\Delta x$. It follows that $\Delta x = \frac{mg}{k}(\sin\alpha - \mu\cos\alpha) \approx 0.35\,\mathrm{m}$ (*1 point*).

b) Once the box had reached the equilibrium state it's initial potential energy had been transformed into potential energy of the spring and heat due to friction (*1 point*). Let h be the distance between the locations of the box in the most compressed state of the spring and in the equilibrium position. Work done to overcome friction can be written as $W_f = \mu mg\cos\alpha\left(\frac{H}{\sin\alpha} + 2h\right)$ (*1 point*) and the law of conservation of energy for the final position as $mgH = k\frac{\Delta x^2}{2} + W_f$ (*1 point*). Both H and h are unknown, so we need another equation that can be written by examining the most compressed state. The law of conservation of energy for that state takes the form $mg(H + h\sin\alpha) = \mu mg\cos\alpha\left(\frac{H}{\sin\alpha} + h\right) + k\frac{(\Delta x + h)^2}{2}$ (*1 point*). The right hand side of the last equation can be rewritten as $\mu mg\cos\alpha\left(\frac{H}{\sin\alpha} + h\right) + k\frac{(\Delta x+h)^2}{2} = W_f - \mu mgh\cos\alpha + k\frac{\Delta x^2}{2} + kh\Delta x + k\frac{h^2}{2}$, so $mgH + mgh\sin\alpha = k\frac{\Delta x^2}{2} + W_f - \mu mgh\cos\alpha + kh\Delta x + k\frac{h^2}{2}$ (*1 point*). Using conservation of energy for the final state we get $mgH + mgh\sin\alpha = mgH - \mu mgh\cos\alpha + kh\Delta x + k\frac{h^2}{2}$. We can also substitute Δx and divide the equation by mg to obtain $h\sin\alpha = h\sin\alpha - 2\mu h\cos\alpha + k\frac{h^2}{2mg}$. Rearranging gives the quadratic equation $h\left(\frac{kh}{2mg} - 2\mu\cos\alpha\right) = 0$ (*1 point*). The two solutions of this equation are $h = 0$, which does not correspond to the case that was observed, and $h = \frac{4\mu mg\cos\alpha}{k}$ (*1 point*). The conservation of energy for the final state can now be written as $mgH = k\frac{\Delta x^2}{2} + \mu mgH\cot\alpha + 8\frac{\mu^2 m^2 g^2\cos^2\alpha}{k}$. Substituting Δx allows us to write $mgH = \frac{m^2g^2}{2k}(\sin\alpha - \mu\cos\alpha)^2 + \mu mgH\cot\alpha + 8\frac{\mu^2m^2g^2\cos^2\alpha}{k}$, which gives us the final result $H = \frac{mg}{k(1-\mu\cot\alpha)}\left(\frac{(\sin\alpha - \mu\cos\alpha)^2}{2} + 8\mu^2\cos^2\alpha\right) \approx 2.13\,\mathrm{m}$ (*1 point*).