The 28th International Science Olympiad for Young Mathematicians, Physicists and Chemists
November 3, 2015
Physics solutions - Form 12

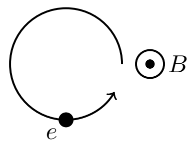
1. a) The block is acted upon by gravity and also by the forces caused by the inside and outside air pressures. In equilibrium $mg + p_0 S = p_1 S$, where p_1 is the inside air pressure, that leads to $m = \frac{p_1 - p_0}{g} S$ (*1 point*). Let $V_0 = SH_0$ and $V_1 = SH_1$ be the initial and final volumes of the air in the tube. Since the tube was not thermally isolated from the environment we can consider it to be isothermal and so $p_0 V_0 = p_1 V_1$ (*1 point*). This allows us to write $m = \left(\frac{H_0}{H_1} - 1\right) \frac{p_0 S}{g} \approx 145\,\mathrm{g}$ (*1 point*).

b) The height of the block is $h = \frac{m}{\rho S} \approx 7.5\,\mathrm{cm}$ (*1 point*).

c) Opening the valve caused the pressures below the plate and outside the cylinder to become equal. Equilibrium of forces in the final state takes the form $mg + p_2 S = p_0 S$ (*1 point*), where p_2 is the new inside air pressure, and $p_2 = \frac{H_0 - H_1 - h}{H_0 - H_2 - h} p_0$ (*1 point*), where $H_0 - H_1 - h$ and $H_0 - H_2 - h$ are the heights of the columns of air above the block when the valve was opened and when the block stopped moving respectively. Combining those two equations we get $p_0 - \frac{mg}{S} = \frac{H_0 - H_1 - h}{H_0 - H_2 - h} p_0$ (*1 point*). From the previous question we know that $\frac{mg}{p_0 S} = \frac{H_0}{H_1} - 1$, so $\frac{H_0 - H_1 - h}{H_0 - H_2 - h} = \frac{2H_1 - H_0}{H_1}$ and $H_2 = H_0 - h - \frac{H_1}{2H_1 - H_0}(H_0 - H_1 - h) \approx 171.4\,\mathrm{cm}$ (*1 point*).

2. a) Without the presence of a magnetic field the centripetal force would be provided only by the electric attraction between the charges. $\frac{mv^2}{R_E} = k_e \frac{Qq}{R_E^2}$ (*1 point*). The speed of the electron is $v = \frac{2\pi R_E}{T}$, so we can write $\frac{4\pi^2 m R_E}{T^2} = k_e \frac{Qq}{R_E^2}$ (*1 point*) and $R_E = \sqrt[3]{k_e \frac{Qq}{4\pi^2 m} T^2} \approx 0.89\,\mathrm{m}$ (*1 point*).

b) For a fixed orbital period centripetal force will be linear with respect to orbital radius, as shown in the answer of the previous question. Since $R > R_E$ the centripetal force is stronger with the magnetic field present (*1 point*). This means that the magnetic and electric forces are in the same direction. The magnetic field must be orientated as depicted in the figure on the right (*1 point*).

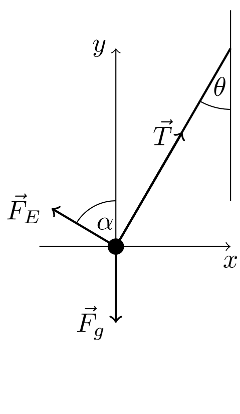


c) In the presence of the magnetic field centripetal force can be written as $\frac{mv^2}{R} = q\left(k_e \frac{Q}{R^2} + Bv\right)$ (*1 point*). Therefore $B = \frac{mv}{Rq} - k_e \frac{Q}{R^2 v}$ (*1 point*). Replacing v gives us $B = \frac{2\pi m}{Tq} - \frac{k_e QT}{2\pi R^3} \approx 11\,\mathrm{\mu T}$ (*1 point*).

3. a) Let up be the positive direction. If the box is moved up by Δx from the equilibrium position, the two springs below the box are both extended by Δx and the one spring above the box is compressed by the same amount. The total force applied by the springs is therefore $F = -2k_2\Delta x - k_1 \Delta x = -9k_1 \Delta x$, the negative value meaning that the force is directed down. This allows us to think of the three springs as a single spring that has stiffness $-\frac{F}{\Delta x} = 9k_1$ (*2 points*). The period of such a harmonic oscillator is $T = 2\pi\sqrt{\frac{m}{9k_1}} = \frac{2\pi}{3}\sqrt{\frac{m}{k_1}} = \frac{T_0}{3}$ (*1 point*). Let N be the number of oscillations made by the box during the time t with three springs attached and let N_0 be the number of oscillations with one spring attached. $T = \frac{T}{3}$ leads to $N = 3N_0$ (*1 point*), so if $N - N_0 = 24$ then $N_0 = 12$ and $T_0 = \frac{t}{N_0} = 5.0\,\mathrm{s}$ (*1 point*). The mass of the box is therefore $m = \frac{T_0^2}{4\pi} k_1 \approx 6.2\,\mathrm{kg}$ (*1 point*).

b) Balancing gravity with the force applied by the springs gives us $-mg - 9k_1 x = 0$ (*2 points*) and $x = -\frac{mg}{9k_1} = -69\,\mathrm{cm}$ (*1 point*). Because $x < 0$, the upper spring is extended and the bottom springs are compressed (*1 point*).

4. a) The ball is acted upon by gravity $\vec{F}_g = m\vec{g}$, electrostatic force $\vec{F}_E = q\vec{E}$ and the tension of the string $\vec{T}$. The equilibrium condition is $\vec{F}_g + \vec{F}_E + \vec{T} = \vec{0}$ (*1 point*). The y-components of the forces give us the equation $qE\sin\alpha = T\sin\theta$ (*1 point*) and the x-components give us $qE\cos\alpha + T\cos\theta = mg$ (*1 point*). From the second equation we get $qE\cos\alpha = mg - T\cos\theta$. This and the equation for the y-components lead to the equations $q^2E^2\cos^2\alpha = m^2g^2 - 2Tmg\cos\theta + T^2\cos^2\theta$ and $q^2E^2\sin^2\alpha = T^2\sin^2\theta$. Adding the respective sides of these equations gives us



$q^2E^2 = m^2g^2 - 2Tmg\cos\theta + T^2$ (*2 points*). Because $qE = mg$ we can rewrite this as $T^2 - 2Tmg\cos\theta = 0$ (*1 point*). $T = 0$ would mean that the ball would always be in equilibrium (*1 point*). The fact that the ball is stationary only if $\theta = 30°$ means that $T = 2mg\cos\theta \approx 17\,\mathrm{N}$ (*1 point*).

b)We can rewrite the equation for the x-components of the forces as $qE\sin\alpha = 2mg\cos\theta\sin\theta = mg\sin(2\theta)$ (*2 points*) and therefore $\alpha = \arcsin\frac{mg\sin(2\theta)}{qE} = 2\theta = 60°$ (*2 points*). $\alpha = 120°$ would also be a solution of the equation $qE\sin\alpha = mg\sin(2\theta)$, but it is known that the vertical component of the electric field is positive, which means that $\alpha < 90°$ (*2 points*).